Dispersion of Parallel-Coupled Microstrip

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Abstract—**Dispersion predictions for the even and odd modes of parallel-coupled microstrip obtained by using the LSE-mode model for microstrip are found to be in good agreement with recently published measurements. Both the basic dispersion relationship and the empirical factor G appear to be the same as for single microstrip lines if the even- and odd-mode impedances used are those of the total parallel-coupled configuration rather than those of a single line of the coupled pair.**

Introduction

Measured dispersion data on the even and odd modes of coupled microstrip lines have recently been published by Gould and Talboys [1], who measured the resonances of rings of concentric pairs of microstrip lines on an alumina substrate 0.025 in thick. These data will be used to verify the applicability of the LSE-mode model [2] of microstrip propagation to coupled microstrip lines.

Dispersion Analysis

A summary of the parameters of the lines described in [1] is given in Table I. Physical dimensions of the coupled lines are shown in Fig. 1. From [2] the dispersion relationships are

$$\epsilon_e = \epsilon_s - \frac{\epsilon_s - \epsilon_{e0}}{1 + G(f^2/f_p^2)} \quad (1)$$

$$f_p = \frac{Z_0}{2\mu H} \quad (2)$$

$$G \approx 0.6 + 0.009 Z_0 \quad (3)$$

where

- ϵ_e the frequency-sensitive effective dielectric constant;
- ϵ_{e0} the effective dielectric constant at zero frequency;
- ϵ_s the substrate dielectric constant;
- Z_0 the characteristic impedance at zero frequency;
- μ 31.92 nH/in;
- G an empirical parameter;
- H the substrate thickness.

In literature on the design of microwave components with parallel-coupled lines, it is conventional to define even-mode impedance Z_{0e} and odd-mode impedance Z_{0o} with reference to a single one of the coupled lines measured to ground, although the actual modes travel on the pair of lines.

In the even mode the two strips are at the same potential, and the total current is twice that on a single strip. Thus the total mode impedance is half that of a single strip, and dispersion for even-mode propagation is computed by substituting $Z_{0e}/2$ for Z_0 in (2) and (3).

In the odd mode the two strips are at opposite potentials, and the voltage between strips is twice that of a single strip to ground. Thus the total mode impedance is twice that of a single strip, and the dispersion for odd-mode propagation is computed by substituting $2Z_{0o}$ for Z_0 in (2) and (3).

The plotted points of guide wavelength presented by Gould and Talboys [1] for the coupled microstrips of Table I were used to compute values of effective dielectric constant. These values are shown by the circles in Fig. 2. For each set of points, a value of effective dielectric constant at zero frequency ϵ_{e0} was estimated, consistent with the requirement that the slope of the dispersion curve must be zero at zero frequency. Values from Table I were substituted into the dispersion relationships, (1)–(3), and the effective dielectric constant was computed at even-numbered frequencies to yield the solid-line curves of Fig. 2. In spite of scatter in the measurements and inaccuracies in the values of substrate dielectric constant, zero-frequency effective dielectric constant, and especially in the mode impedances, Fig. 2 shows very good agreement between the LSE model prediction and the measured values.

For the even-mode characteristics of lines 1*b* and 2*b* in Fig. 2, it is possible to find the factor G by using the inflection-point method described in [2], and to compare it with the value of G given by the empirical formula, (3). (The inflection points of the other characteristics are beyond the plotted range.) From [2],

$$\epsilon_{ei} = \tfrac{1}{4}(\epsilon_s + 3\epsilon_{e0}) \quad (4)$$

$$f_i = \frac{f_p}{\sqrt{3G}} \quad (5)$$

where ϵ_{ei} is the value of effective dielectric constant at the inflection point, which occurs at frequency f_i. Table II gives the values of G found by using the two methods. Because of uncertainty in the dispersion curves, the values of G obtained by using the inflection method are approximations dependent on the estimations of f_i at ϵ_{ei}. The significant point is that both methods give about the same values.

Conclusion

The agreement between predictions and measurements has shown that the LSE-mode microstrip dispersion model and the associated factor G, which were derived from consideration of single microstrip

Manuscript received June 2, 1972; revised August 3, 1972. This short paper is based upon work performed at COMSAT Laboratories under Corporate sponsorship.
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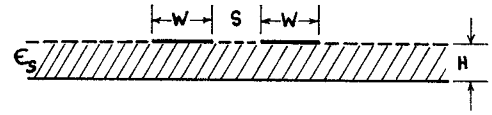

Fig. 1. Parallel-coupled microstrip.

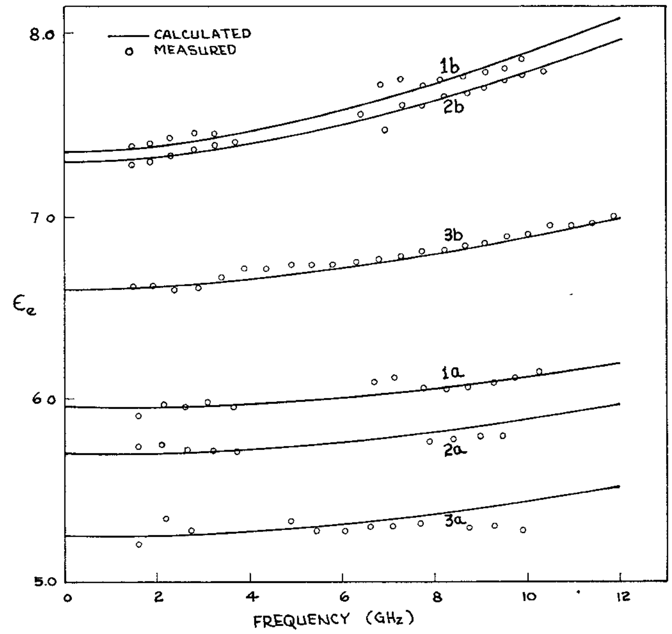

Fig. 2. Measured and calculated dispersion of coupled microstrip.

TABLE I
PARAMETERS OF MEASURED PARALLEL-COUPLED MICROSTRIP

Line	Mode	Approximate[a] Impedance	Calculated[a] ϵ_{e0}	Measured[b] ϵ_{e0}	W/H	S/H
1*a*	odd	46.8	5.95	5.95	0.86	1.12
1*b*	even	59.4	7.14	7.4		
2*a*	odd	44.0	5.80	5.7	0.80	0.69
2*b*	even	64.6	7.13	7.3		
3*a*	odd	46.6	5.54	5.25	0.30	0.19
3*b*	even	110.9	6.40	6.6		

[a] Calculated by the MSTRIP [3] program using $\epsilon_s = 10.0$.
[b] Estimated by extrapolation of measured dispersion to zero frequency.

TABLE II

FACTOR G BY TWO METHODS

				G	
Line	Mode	ϵ_{ei}	f_i	Inflection Point	Equation (3)
$1b$	even	8.0	11.4	0.87	0.87
$2b$	even	8.0	12.8	0.835	0.89

lines, also hold for the even and odd modes of parallel-coupled microstrip lines if total mode impedances are employed in the formulas.

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Effects of the T-Junction Discontinuity on the Design of Microstrip Directional Couplers

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Abstract—**The influence of the T-junction effect on the microstrip 3-dB branchline and rat-race directional couplers design has been described. In the proposed method, the T-junction equivalent circuit similar to the circuit used by Franco and Oliner [1] for the symmetric stripline junction is applied. Simple design expressions that combine the modified characteristic impedance and electrical length of the coupler arm with the equivalent parameters of the T junction are given. These expressions are helpful in the determination of the microstrip coupler dimensions. The experimental verification of an improvement of the design procedure is also included.**

In a paper by Leighton and Milnes [2], the influence of T-junction reactances on the characteristics of 3-dB branch line and rat-race couplers has been considered. The method proposed in [2] was based on the T-junction equivalent circuit used by Altschuler and Oliner [3]. In the practical design of all microwave devices containing the T junctions, it is sometimes more convenient to represent this discontinuity by the equivalent circuit in the form recommended by Franco and Oliner [1] (Fig. 1). Knowing the parameters of this equivalent circuit, we can determine the correct way of finding the main dimensions of the devices that contain the T junction.

In order to design the 3-dB branch line and rat-race directional couplers, the calculations based on the procedure recommended by Franco and Oliner were carried out, with the difference being that the effective linewidth D was determined with the same procedure as by Leighton and Milnes [2]. The results obtained in the range of impedance ratios usually met (in the case of 3-dB branch line and rat-race couplers) are presented in a graphical form (Figs. 2 and 3) showing the dependence of the circuit parameters on the geometrical dimensions of the lines involved and on the frequencies used. As mentioned earlier, the parameters can be applied to the design of some branch and ring-type 3-dB couplers. This design is possible when one compares two equivalent circuits, of which the first represents the ideal coupler without junction reactances and the second corresponds to the coupler, including the parameters of the T junction. These equivalent circuits can be analyzed by the even-and-odd excitation method employed by Reed and Wheeler [4]. Determining and comparing the wave transmission matrix for both cases, we can evaluate the modified characteristic impedance Z_m of the branch (stub arm) and its electrical length Θ_m between the reference planes. These new parameters compensate discontinuity effects of the T junction. For

Manuscript received June 2, 1972; revised August 17, 1972.
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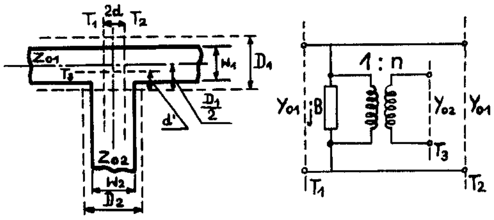

Fig. 1. Equivalent circuit for microstrip T junction.

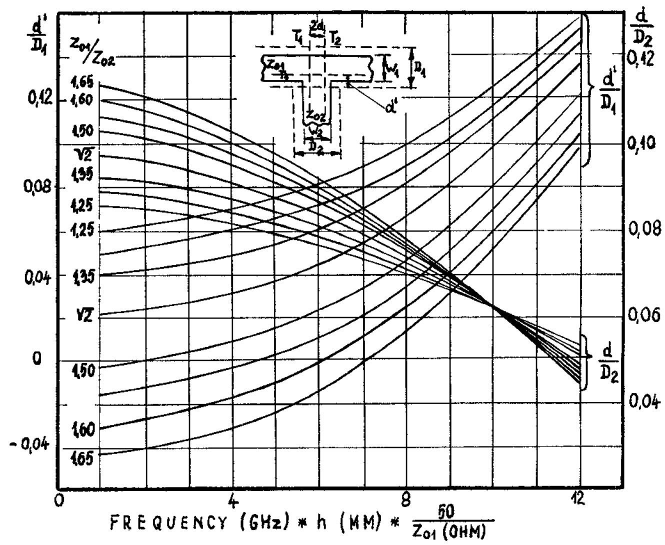

Fig. 2. Reference plane shifts in the main and stub arms as a function of frequency f and substrate thickness h.

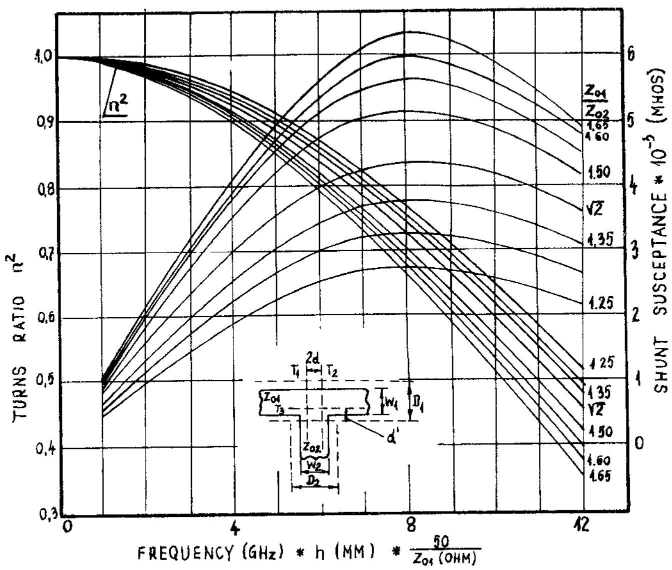

Fig. 3. Transformer turn ratio n and shunt susceptance B as a function of frequency f and substrate thickness h.

the case of the branch line coupler, they are given by

$$Z_m = \frac{n^2 Z_i}{\sqrt{1 - (BZ_i)^2}}$$

$$\Theta_m = \arccos (BZ_i) \tag{1}$$

where

Z_i characteristic impedance of branch (stub arm) for the ideal case without junction effects (for the 3-dB coupler, $Z_i = Z_0/\sqrt{2}$);